

hep-th/0101165
December 2000

Supersymmetry Transformation of Quantum Fields II: Supersymmetric Yang-Mills Theory

Christian Rupp ¹
Institut für Theoretische Physik
Universität Bern
Sidlerstrasse 5
CH - 3012 Bern
Switzerland

Klaus Sibold ²
Institut für Theoretische Physik
Universität Leipzig
Augustusplatz 10/11
D - 04109 Leipzig
Germany

Abstract

We study the transformation law of quantum fields in super Yang-Mills theory quantized in the Wess-Zumino gauge. It can be derived from a local version of generalized Slavnov-Taylor identities for general Green functions. Under suitable normalization conditions the transformations are local. Within the vector multiplet anomalous dimensions become equal. The breaking of susy shows up in Fock but not in Hilbert space and is not reflected in the transformation law for the fields.

PACS numbers: 03.70.+k, 11.15.Bt, 11.30.Pb

Keywords: Quantum Field Theory, Supersymmetry, Wess-Zumino Gauge,
Anomalous Dimension.

¹Email: rupp@itp.unibe.ch

²Email: Klaus.Sibold@itp.uni-leipzig.de

1 Introduction

The quantization of $N = 1$ supersymmetric Yang-Mills theory (SYM) can be performed in essentially two ways: one is the linear realization of supersymmetry in terms of superfields, the other one uses the elimination of longitudinal and auxiliary fields and leads to non-linear realization of supersymmetry (the Wess-Zumino gauge). In the linear realization one is faced with an off-shell infrared problem since the vector superfield is dimensionless. To overcome it one breaks susy explicitly and can guarantee existence and susy covariance only for Green functions of gauge (i.e. BRS) invariant operators [1]. Similarly in the non-linear version: at most Green functions of BRS invariant quantities possess proper covariance under susy because gauge fixing terms usually break supersymmetry. Although the treatment of the renormalization procedure in terms of the vertex functional seems to be in good shape [2, 3, 4, 5, 6] it is not clear how a supersymmetry charge can be defined and how it acts on the fields. We investigated this question in the context of the Wess-Zumino model without auxiliary fields and in supersymmetric QED (in the Wess-Zumino gauge) [7]. It turned out that the Wess-Zumino model admits a susy charge which generates the non-linear field transformation and commutes with the S-matrix i.e. is conserved. In SQED however the susy charge Q_α becomes time-dependent according to

$$Q_\alpha^{\text{out}} - Q_\alpha^{\text{in}} = \left[Q^{\text{BRS}}, \frac{\partial \Gamma_{\text{eff}}}{\partial \varepsilon^\alpha} \right] \quad (1.1)$$

where Q^{BRS} denotes the BRS charge and $\partial \Gamma_{\text{eff}} / \partial \varepsilon$ arises from the gauge fixing term. The field transformations are local, but for $\lambda, \bar{\lambda}$ (photino) not consistent with time evolution. It turns out however that the combinations $\lambda - \varepsilon \bar{c}$, $\bar{\lambda} - \bar{\varepsilon} c$ – here \bar{c} is the antighost field – indeed have a transformation law which is consistent with time evolution.

In the present paper we study the analogous situation for SYM where the complication arises that B – the Lagrange multiplier field showing up in gauge fixing – is no longer a free field. This difficulty can be overcome by a suitable choice of normalization conditions, as will be explained in sect. 3. This normalization affects the anomalous dimension of λ ($\bar{\lambda}$) such that it becomes equal to that of the vector.

2 The model and its vertex functional

We work with SYM, assume a simple gauge group and matter multiplets in some representation. In order to make the paper self-contained we write down explicitly the field

transformations in the classical approximation

$$sA_\mu = \partial_\mu c - ig[c, A_\mu] + i\varepsilon\sigma_\mu\bar{\lambda} - i\lambda\sigma_\mu\bar{\varepsilon} - i\omega^\nu\partial_\nu A_\mu, \quad (2.1)$$

$$s\lambda^\alpha = -ig\{c, \lambda^\alpha\} + \frac{i}{2}(\varepsilon\sigma^{\rho\sigma})^\alpha F_{\rho\sigma} + i\varepsilon^\alpha D - i\omega^\nu\partial_\nu\lambda^\alpha, \quad (2.2)$$

$$s\bar{\lambda}_{\dot{\alpha}} = -ig\{c, \bar{\lambda}_{\dot{\alpha}}\} - \frac{i}{2}(\bar{\varepsilon}\bar{\sigma}^{\rho\sigma})_{\dot{\alpha}} F_{\rho\sigma} + i\bar{\varepsilon}_{\dot{\alpha}} D - i\omega^\nu\partial_\nu\bar{\lambda}_{\dot{\alpha}}, \quad (2.3)$$

$$s\phi_i = -igc\phi_i + \sqrt{2}\varepsilon\psi_i - i\omega^\nu\partial_\nu\phi_i, \quad (2.4)$$

$$s\phi_i^\dagger = +ig(\phi_i^\dagger c)_i + \sqrt{2}\bar{\psi}_i\bar{\varepsilon} - i\omega^\nu\partial_\nu\phi_i^\dagger, \quad (2.5)$$

$$s\psi_i^\alpha = -igc\psi_i^\alpha + \sqrt{2}\varepsilon^\alpha F_i - \sqrt{2}i(\bar{\varepsilon}\bar{\sigma}^\mu)^\alpha D_\mu\phi_i - i\omega^\nu\partial_\nu\psi_i^\alpha, \quad (2.6)$$

$$s\bar{\psi}_{i\dot{\alpha}} = -ig(\bar{\psi}_{i\dot{\alpha}}c)_i - \sqrt{2}\bar{\varepsilon}_{\dot{\alpha}} F_i^\dagger + \sqrt{2}i(\varepsilon\sigma^\mu)_{\dot{\alpha}}(D_\mu\phi_i)^\dagger - i\omega^\nu\partial_\nu\bar{\psi}_{i\dot{\alpha}} \quad (2.7)$$

$$sc = -igc^2 + 2i\varepsilon\sigma^\nu\bar{\varepsilon}A_\nu - i\omega^\nu\partial_\nu c, \quad (2.8)$$

$$s\varepsilon^\alpha = 0, \quad (2.9)$$

$$s\bar{\varepsilon}^{\dot{\alpha}} = 0, \quad (2.10)$$

$$s\omega^\nu = 2\varepsilon\sigma^\nu\bar{\varepsilon}. \quad (2.11)$$

(We use the conventions of [6].) The auxiliary fields D and F_i have to be replaced by the equations of motion. The ordinary transformation parameters have been promoted to constant ghost fields ($\varepsilon, \bar{\varepsilon}$ susy; ω^ν translations).

The classical action

$$\Gamma_{\text{cl}} = \Gamma_{\text{inv}} + \Gamma_{\text{g.f.}} \quad (2.12)$$

$$\Gamma_{\text{inv}} = \int \text{tr} \left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\not{D}\bar{\lambda} \right) + \text{matter} \quad (2.13)$$

$$\Gamma_{\text{g.f.}} = s \int \text{tr} \left(\frac{\xi}{2}\bar{c}B + \bar{c}\partial A \right) \quad (2.14)$$

is invariant under the generalized BRS transformations. In order to deal with the non-linearity of the transformations and the fact that they close only on-shell one introduces external fields and lets the action depend upon them even bilinearly:

$$\Gamma = \Gamma_{\text{cl}} + \Gamma_{\text{g.f.}} + \Gamma_{\text{ext.f.}} + \Gamma_{\text{bil}} \quad (2.15)$$

$$\Gamma_{\text{ext.f.}} = \int \left(\text{tr} \left(Y_{A^\mu} sA^\mu + Y_\lambda s\lambda + Y_{\bar{\lambda}} s\bar{\lambda} + Y_c sc \right) + (Y_\phi s\phi + Y_\psi s\psi + c.c.) \right) \quad (2.16)$$

$$\Gamma_{\text{bil}} = \int \frac{1}{2} \text{tr} (Y_\lambda \varepsilon + Y_{\bar{\lambda}} \bar{\varepsilon})^2 + (Y_\psi \varepsilon + Y_{\bar{\psi}} \bar{\varepsilon})^2. \quad (2.17)$$

The classical vertex functional Γ (2.15) satisfies the ST identity

$$(\Gamma) \equiv \sum_\phi \int \frac{\delta\Gamma}{\delta Y_\phi} \frac{\delta\Gamma}{\delta\phi} + \sum_{\phi'} (s\phi') \frac{\delta\Gamma}{\delta\phi'} = 0 \quad (2.18)$$

(here ϕ runs over all non-linearly transforming fields, ϕ' over all linearly transforming ones).

It has been shown in [2, 4, 5] that the gauge condition

$$\frac{\delta\Gamma}{\delta B} = \xi B + \partial A, \quad (2.19)$$

the rigid Ward identity

$$W\Gamma = \sum_{\text{all fields } f} \int i \delta f \frac{\delta\Gamma}{\delta f} = 0, \quad (2.20)$$

the translational ghost equation

$$\frac{\partial\Gamma}{\partial\omega^\nu} = \frac{\partial\Gamma_{\text{ext}}}{\partial\omega^\nu} \quad (2.21)$$

and the ST identity (2.18) determine the vertex functional uniquely, if the Adler-Bardeen anomaly is absent and one has supplied suitable normalization conditions.

The aim is now to study the transformation law of the quantum fields under the above transformations.

3 Transformations of the quantum fields

In the perturbative context in which we are working operator equations can be deduced by using the LSZ reduction formalism, hence we go over to Z , the generating functional of general Green functions. The translational ghost eqn. (2.21) leads to the simple WI

$$\sum_f \int \partial_\nu f \frac{\delta\Gamma}{\delta f} = 0 \quad (3.1)$$

(sum over all fields) which expresses translational invariance of Γ . On Z the WI reads

$$\sum_f \int \partial_\nu j_f \frac{\delta Z}{\delta j_f} = 0. \quad (3.2)$$

Rendering the transformations local introduces the energy-momentum-tensor

$$\sum_f \partial_\nu j_f \frac{\delta Z}{\delta j_f} = [\partial^\mu T_{\mu\nu}] \cdot Z. \quad (3.3)$$

For a string X of propagating fields the respective relation for Green functions is

$$\sum_\phi \langle \partial_\nu \delta(x-y) \phi(x) X_{\tilde{\phi}} \rangle = \langle \partial^\mu T_{\mu\nu}(x) X \rangle \quad (3.4)$$

($\check{\phi}$ indicates that ϕ is missing in the string). LSZ reduction w.r.t. X yields zero on the l.h.s., on the r.h.s. the matrix element defined by X , hence conservation of the energy-momentum operator

$$0 = \partial^\mu T_{\mu\nu}^{\text{Op}}. \quad (3.5)$$

Reducing w.r.t. $X_{\check{\phi}}$, integrating over all of space $\int d^3x$ and the time slice $(x_0 - \mu, x_0 + \mu)$ brings about

$$\partial_\nu \varphi^{\text{Op}} = i [P_\nu, \varphi]^{\text{Op}}, \quad (3.6)$$

i.e. the standard transformation of a quantum field operator under translations ($\varphi = A_\mu, \lambda, \bar{\lambda}, B, c, \bar{c}, \phi, \phi^\dagger, \psi, \psi^\dagger$).

The BRS charge and transformation properties can be derived from a version of the ST identity

$$\left(j_{\bar{c}} \frac{\delta Z}{\delta j_B} + \sum_\varphi j_\varphi \frac{\delta Z}{\delta Y_\varphi} \right) \Big|_{\varepsilon=\bar{\varepsilon}=\omega^\nu=0} = [\partial^\mu J_\mu^{\text{BRS}}] \cdot Z \Big|_{\varepsilon=\bar{\varepsilon}=\omega^\nu=0} \quad (3.7)$$

in which the gauge BRS transformation has been made to a local one. For Green functions determined by X this eqn. takes the form

$$\delta(x-y) \langle B(x) X_{\bar{c}} \rangle + \sum_\varphi \delta(x-y) \left\langle \frac{\delta \Gamma_{\text{eff}}}{\delta Y_\varphi(x)} X_{\check{\varphi}} \right\rangle = \langle \partial^\mu J_\mu^{\text{BRS}}(x) X \rangle \quad (3.8)$$

(all at $\varepsilon = \bar{\varepsilon} = \omega = 0$). Like for the translations it follows first that the BRS charge

$$Q^{\text{BRS}} = \int d^3x J_0^{\text{BRS}} \quad (3.9)$$

is conserved (reduction w.r.t. X), then for the fields that they transform as

$$i[Q^{\text{BRS}}, \bar{c}]^{\text{Op}} = B^{\text{Op}} \quad (3.10a)$$

$$i[Q^{\text{BRS}}, \varphi]^{\text{Op}} = \frac{\delta \Gamma_{\text{eff}}}{\delta \varphi}^{\text{Op}} \quad (3.10b)$$

($\varphi = A_\mu, \lambda, \bar{\lambda}, B, c, \phi, \phi^\dagger, \psi, \psi^\dagger$). In order to derive the susy transformations we assume all parameters $\varepsilon, \bar{\varepsilon}, \omega$ to be local:

$$\begin{aligned} \text{loc} Z &\equiv i j_{A\mu} \frac{\delta Z}{\delta Y_{A\mu}} + j_{A\mu} \omega^\nu \partial_\nu \frac{\delta Z}{\delta j_{A\mu}} - i j_c \left(2i\varepsilon \sigma^\nu \bar{\varepsilon} \frac{\delta Z}{\delta j_A^\nu} - i\omega^\nu \partial_\nu \frac{\delta Z}{\delta j_c} \right) \\ &\quad - i j_{\bar{c}} \left(\frac{\delta Z}{\delta j_B} - i\omega^\nu \partial_\nu \frac{\delta Z}{\delta j_{\bar{c}}} \right) + i j_B \left(2i\varepsilon \sigma^\nu \bar{\varepsilon} \partial_\nu \frac{\delta Z}{\delta j_{\bar{c}}} - i\omega^\nu \partial_\nu \frac{\delta Z}{\delta j_B} \right) \\ &\quad - i\eta_\lambda^\alpha \frac{\delta Z}{\delta Y_\lambda^\alpha} - i\bar{\eta}_{\lambda\dot{\alpha}} \frac{\delta Z}{\delta \bar{Y}_{\lambda\dot{\alpha}}} - i j_{\phi_i} \frac{\delta Z}{\delta Y_{\phi_i}} - i j_{\phi_i}^\dagger \frac{\delta Z}{\delta Y_{\phi_i}^\dagger} \\ &\quad - i\eta_{\psi_i}^\alpha \frac{\delta Z}{\delta Y_{\eta_i}^\alpha} - i\bar{\eta}_{\psi_i\dot{\alpha}} \frac{\delta Z}{\delta \bar{Y}_{\psi_i\dot{\alpha}}} - 2i\varepsilon \sigma^\mu \bar{\varepsilon} \frac{\delta Z}{\delta \omega^\mu} \\ &= [\partial^\mu J_\mu^{\text{BRS}} + \partial^\mu \varepsilon^\alpha K_{\mu\alpha} + \partial^\mu \bar{\varepsilon}_{\dot{\alpha}} \bar{K}_{\mu}^{\dot{\alpha}} + \partial^\mu \omega^\nu K_{\mu\nu}] \cdot Z \end{aligned} \quad (3.11)$$

$(\eta_\lambda, \bar{\eta}_\lambda, \eta_{\psi_i}, \bar{\eta}_{\psi_i} \text{: sources for } \lambda, \bar{\lambda}, \psi_i, \bar{\psi}_i).$

J_μ depends on the ghosts $\varepsilon, \bar{\varepsilon}, \omega$ and for constant ghosts integration yields zero on the r.h.s.

For quite a few steps the calculation runs completely parallel to the abelian case [7], hence we can be very brief. Differentiating (3.11) w.r.t. $\varepsilon^\alpha(z)$, the local susy ghost, (analogously for $\bar{\varepsilon}_\alpha(z)$) and performing LSZ reduction we obtain the operator equation

$$0 = \frac{\delta}{\delta \varepsilon^\alpha(z)} \partial^\mu J_\mu^{\text{Op}}(x) + i \text{T} \left(\partial^\mu J_\mu(x) \frac{\delta \Gamma_{\text{eff}}}{\delta \varepsilon^\alpha(z)} \right)^{\text{Op}} + \partial_x^\mu \delta(x-z) K_{\mu\alpha}^{\text{Op}}(x). \quad (3.12)$$

Integration over z yields

$$0 = \partial_x^\mu \partial_{\varepsilon^\alpha} J_\mu^{\text{Op}}(x) + i \int d^4 z \text{T} \left(\partial^\mu J_\mu(x) \frac{\delta \Gamma_{\text{eff}}}{\delta \varepsilon^\alpha(z)} \right)^{\text{Op}}, \quad (3.13)$$

i.e. a susy current which is not conserved. Similarly the susy charge defined by

$$Q_\alpha(t) \equiv - \int d^3 x \partial_{\varepsilon^\alpha} J_0^{\text{Op}}(x) \quad (3.14)$$

will depend on t . Integrating (3.13) over all of x -space leads to

$$0 = \int dt \partial_t Q_\alpha(t) - i [Q^{\text{BRS}}, \partial_{\varepsilon^\alpha} \Gamma_{\text{eff}}]^{\text{Op}} \quad (3.15)$$

Taking the time integral for asymptotic times $t = \pm\infty$ and identifying there the charges we can write

$$Q_\alpha^{\text{out}} - Q_\alpha^{\text{in}} = i [Q^{\text{BRS}}, \partial_{\varepsilon^\alpha} \Gamma_{\text{eff}}]^{\text{Op}}. \quad (3.16)$$

Since Q_α^{out} develops out of Q_α^{in} via the time evolution operator S , the scattering operator,

$$Q_\alpha^{\text{out}} = S Q_\alpha^{\text{in}} S^\dagger, \quad (3.17)$$

(3.16) implies

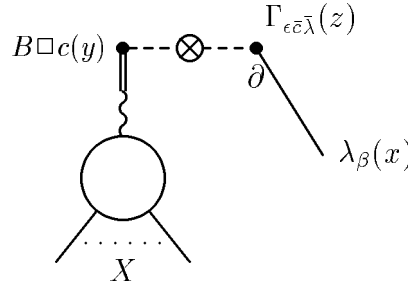
$$[Q_\alpha^{\text{in}}, S] = -i [Q^{\text{BRS}}, \partial_{\varepsilon^\alpha} \Gamma_{\text{eff}} \cdot S]. \quad (3.18)$$

Here we have used that Q^{BRS} and S commute. The interpretation of this result is clear: the charge Q_α^{in} which may be taken to be the generator of susy transformations on the free in-states does not commute with the S -operator, the reason being the ε -dependence of Γ_{eff} . Looking at (2.14) this arises from the gauge fixing term. Matrix elements between physical states however yield a vanishing r.h.s. in (3.18), hence there Q_α^{in} is a conserved charge.

The transformation law for (non-linearly transforming) fields φ can be found by differentiating (3.11) w.r.t. ε and φ . After LSZ reduction one obtains the operator relation

$$\begin{aligned}
& -\delta(y-x) \frac{\delta^2 \Gamma_{\text{eff}}}{\delta Y(y) \delta \varepsilon^\alpha(z)} - i\delta(y-x) T \left(\frac{\delta \Gamma_{\text{eff}}}{\delta Y(y)} \frac{\delta \Gamma_{\text{eff}}}{\delta \varepsilon^\alpha(z)} \right) \\
& = iT \left(\frac{\delta}{\delta \varepsilon^\alpha(z)} \partial^\mu J_\mu(y) \varphi(x) \right) - T \left(\partial^\mu J_\mu(y) \frac{\delta \Gamma_{\text{eff}}}{\delta \varepsilon^\alpha(z)} \varphi(x) \right) + i\partial_y^\mu \delta(y-x) T(K_{\mu\alpha}(y) \varphi(x))
\end{aligned} \tag{3.19}$$

Since in the T-product $T(\partial J \cdot \partial_\varepsilon \Gamma_{\text{eff}} \cdot \varphi)$ distributional singularities may arise for coinciding points one cannot straightforwardly integrate (3.19) but has to determine these singularities. As a consequence of the renormalization scheme they are well-defined but have to be identified. This analysis parallels first again the abelian case: contributions proportional to $\delta(y-x)$ arise and are cancelled with the operator product $T(\delta \Gamma_{\text{eff}} / \delta Y(y) \cdot \delta \Gamma_{\text{eff}} / \delta \varepsilon^\alpha(z))$ on the l.h.s. But then occurs a change: the contributions which lead to a double delta function $\delta(x-y)\delta(y-z)$ are represented by diagrams



$$\tag{3.20}$$

which differ from SQED in that the ghost propagator $\Delta_{c\bar{c}}$ is no longer a free one and in that the vertex function $\Gamma_{\varepsilon \bar{c} \bar{\lambda}}$ receives loop corrections because the ghost equation of motion is non-trivial. This implies that the local contribution $a\varepsilon B$ to the transformation $s\lambda$ becomes order dependent through a coefficient a . A way out of this difficulty is to impose as normalization condition

$$(p^2 \Delta_{c\bar{c}}) (\partial_p \Gamma_{\varepsilon \bar{c} \bar{\lambda}}) \Big|_{p=0, s=1} = 1. \tag{3.21}$$

As a consequence of the ghost equation

$$\frac{\delta \Gamma}{\delta \bar{c}} + \partial^\mu \frac{\delta \Gamma}{\delta Y_A^\mu} + i\omega^\nu \partial_\nu \frac{\delta \Gamma}{\delta B} + (2i\varepsilon \sigma^\nu \bar{\varepsilon} \partial_\nu \bar{c} - i\omega^\nu \partial_\nu B) \xi = 0, \tag{3.22}$$

one finds

$$\Gamma_{\varepsilon \bar{c} \bar{\lambda}} = -\partial^\mu \frac{\partial}{\partial \varepsilon} \frac{\delta}{\delta Y_A^\mu} \frac{\delta \Gamma}{\delta \bar{\lambda}} \tag{3.23}$$

$$= -\partial^\mu \left(\left[\frac{\partial}{\partial \varepsilon} \frac{\delta}{\delta Y_A^\mu} \Gamma_{\text{eff}} \right] \cdot \Gamma \right)_{\bar{\lambda}} - \partial^\mu \left(\left[\frac{\delta \Gamma_{\text{eff}}}{\delta Y_A^\mu} \right] \cdot [\partial_\varepsilon \Gamma_{\text{eff}}] \cdot \Gamma \right)_{\bar{\lambda}}. \tag{3.24}$$

The first term on the r.h.s. consists of purely local contributions whereas the second (a double insertion) is non-local. Hence one can rewrite (3.21) as

$$(\sqrt{z_{c\bar{c}}})^2 \left(a_2 + \left(\left[\frac{\delta \Gamma_{\text{eff}}}{\delta Y_A^\mu} \right] \cdot \left[\frac{\partial \Gamma_{\text{eff}}}{\partial \varepsilon} \right] \cdot \Gamma \right) \Big|_{\bar{\lambda}} \Big|_{p=0, s=1} \right) = 1, \quad (3.25)$$

i.e. one fixes indeed the coefficient a_2 of the counterterm $\int \bar{c} \partial (i\varepsilon \sigma \bar{\lambda} - i\lambda \sigma \bar{\varepsilon})$ by this condition:

$$a_2 = 1 + o(\hbar) \quad (3.26)$$

From tree approximation and with the help of the symmetric differential operators (see next section) one reads off that it belongs to the same invariant as $i\lambda \not{\partial} \bar{\lambda}$, the kinetic term. Imposing this normalization condition one can attribute the double delta function contribution of diagram (3.20) to a local transformation

$$s\lambda = \varepsilon B + \dots \quad (3.27)$$

(analogously for $\bar{\lambda}$).

With this result one arrives at the same conclusions as in the abelian situation. The term εB in the transformation law of λ removes the mismatch between the transformation as given by the above Q_α (which has been constructed without contribution of the gauge fixing term) and the time evolution of λ as given by its equation of motion to which the gauge fixing term, of course, contributes. Hence we have local and evolution compatible transformation laws for all fields. The breaking of supersymmetry caused by the gauge fixing is entirely cast into the time dependence of the susy charge.

4 The anomalous dimensions within the vector multiplet

We shall now show that the normalization condition (3.21) has an interesting consequence for the anomalous dimensions within the vector multiplet.

The renormalization group (RG) or Callan-Symanzik (CS) equation which are identical in the massless case follow as usual [1] as differential equations symmetric with respect to the ST identity and compatible with the gauge condition. A basis of the respective

differential operators is provided by

$$_A \equiv \int A \frac{\delta}{\delta A} - Y_A \frac{\delta}{\delta Y_A} - B \frac{\delta}{\delta B} + \bar{c} \frac{\delta}{\delta \bar{c}} \quad (4.28)$$

$$_\lambda \equiv \int \lambda \frac{\delta}{\delta \lambda} - Y_\lambda \frac{\delta}{\delta Y_\lambda} + c.c. \quad (4.29)$$

$$_c \equiv \int -c \frac{\delta}{\delta c} + Y_c \frac{\delta}{\delta Y_c} \quad (4.30)$$

$$_\phi \equiv \int \phi \frac{\delta}{\delta \phi} - Y_\phi \frac{\delta}{\delta Y_\phi} + c.c. \quad (4.31)$$

$$_\psi \equiv \int \psi \frac{\delta}{\delta \psi} - Y_\psi \frac{\delta}{\delta Y_\psi} + c.c. \quad (4.32)$$

The CS operator can be expanded in this basis and the CS eqn. then reads

$$\Gamma \equiv \left(m \partial_m + \beta_g \partial_g - \sum_{\varphi} \gamma_{\varphi \varphi} \right) \Gamma = 0 \quad (4.33)$$

(m comprises all mass parameters, g all couplings of the theory, φ runs over $A, \lambda, c, \phi, \psi$). This implies that apriori the anomalous dimension γ_A of the vector field is not related to γ_λ , the anomalous dimension of $\lambda, \bar{\lambda}$.

Testing (4.33) w.r.t. c and \bar{c} we first find

$$(m \partial_m + \beta_g \partial_g) \Gamma_{c\bar{c}} = (\gamma_A - \gamma_c) \Gamma_{c\bar{c}}, \quad (4.34)$$

hence

$$(m \partial_m + \beta_g \partial_g) \Delta_{c\bar{c}} = (\gamma_c - \gamma_A) \Delta_{c\bar{c}}. \quad (4.35)$$

Similarly,

$$(m \partial_m + \beta_g \partial_g) \Gamma_{\varepsilon \bar{c} \bar{\lambda}} = (\gamma_A - \gamma_\lambda) \Gamma_{\varepsilon \bar{c} \bar{\lambda}} = 0. \quad (4.36)$$

Hence, acting with $(m \partial_m + \beta_g \partial_g)$ on (3.21) we obtain

$$(\gamma_c - \gamma_A) + (\gamma_A - \gamma_\lambda) = 0, \quad (4.37)$$

i.e.

$$\gamma_\lambda = \gamma_c. \quad (4.38)$$

The normalization condition (3.21) leads to coinciding anomalous dimensions for the gaugino and ghost fields. Even more can be said about the anomalous dimensions when we choose the Landau gauge $\alpha = 0$. In this gauge, an additional ghost equation [3] holds,

$$\Gamma \Big|_{\text{ext.f.}=0} = 0, \quad (4.39)$$

$$= \int \left(\frac{\delta}{\delta c} + \text{ig} \left[\bar{c}, \frac{\delta}{\delta B} \right] \right). \quad (4.40)$$

By acting with the commutation relation

$$[,] = (\gamma_c - 2\gamma_A) \int \frac{\delta}{\delta c} \quad (4.41)$$

on Γ , we obtain

$$\gamma_c = 2\gamma_A, \quad (4.42)$$

and together with (4.38)

$$\gamma_\lambda = 2\gamma_A. \quad (4.43)$$

In an arbitrary gauge, (4.42) may be imposed as a normalization condition, fixing the counterterm $\int \text{tr}(cY_c)$.

The result (4.43) fits to the asymptotic supersymmetry which also followed as a consequence of it as shown in the last section. It is not in contradiction to previous calculations [8] because there other normalization conditions have been used and anomalous dimensions depend quite generally on normalization conditions and gauge fixing.

Without further inquiries nothing can be said about the anomalous dimensions within the matter multiplets. It may however very well turn out that a suitable non-linear gauge fixing could render them equal as well. That would be of importance in models with $N = 2$ symmetry. For the $N = 4$ theory, too, the role of gauge fixing and its effect on anomalous dimensions seems not to have been discussed thoroughly.

5 Discussion and conclusions

In the context of $N = 1$ SYM theory we studied the transformation properties of quantum fields in perturbation theories. The generalized ST identity (3.11) with local ghosts for translations, BRS and susy transformations governs these transformation laws via LSZ reduction. As already remarked in [7] this implies representation dependence of operator equations whose validity outside of perturbation theory is not clear.

For translations and BRS we found the standard transformations (3.6), (3.10) which is not surprising since both are conserved. Since the gauge fixing breaks susy the respective charge is not conserved in Fock space but only in the Hilbert space of the theory. It turns out that the vectorino fields $\lambda, \bar{\lambda}$ receive additional contributions in their transformation law. They can be understood as proper, local transformations $\varepsilon B, \bar{\varepsilon} B$ if one imposes a suitable normalization condition (3.21) which determines the relevant part of gauge fixing. For the fields $\lambda - \varepsilon \bar{c}, \bar{\lambda} - \bar{\varepsilon} c$ one then has local susy transformations such that asymptotically susy is restored. This normalization condition also implies that the anomalous dimension of λ equals that of the ghost, and in the Landau gauge (or by imposing an additional normalization condition) it is related to the anomalous dimension of the vector field A_μ .

A comment is in order as to the validity of our derivation. We have written all formulae for a completely massless theory. Then, of course, there is no S-matrix and one cannot go on-shell. Hence we may add mass terms for all fields and thus maintain in general only translational invariance but neither ordinary BRS nor susy. Our equations should then be read modulo soft breaking terms – which is sufficient for clarifying the role of gauge fixing and normalization condition (3.21). For models where susy is maintained but the gauge invariance is completely broken down such that all vector fields (and the vectorinos) get mass our results should be strictly true. Alternatively one could have derived within the massless theory the desired operator equations by performing the reduction not with respect to the asymptotic states but with suitable other combinations.

References

- [1] O. Piguet, K. Sibold, *Renormalized Supersymmetry. The Perturbation Theory of $N = 1$ Supersymmetric Theories in Flat Space-Time*. Birkhäuser, Boston 1986.
- [2] P.L. White, *An analysis of the cohomology structure of super Yang-Mills coupled to matter*, Class. Quantum Grav. **9** (1992) 1663,
P.L. White, *Analysis of the superconformal cohomology structure of $N = 4$ super Yang-Mills*, Class. Quantum Grav. **9** (1992) 413.
- [3] N. Maggiore, O. Piguet and S. Wolf, *Algebraic renormalization of $N = 1$ supersymmetric gauge theories*, Nucl. Phys. **B458** (1996) 403, hep-th/9507045.
- [4] N. Maggiore, O. Piguet and S. Wolf, *Algebraic Renormalization of $N = 1$ Supersymmetric Gauge Theories with Supersymmetry Breaking Masses*, Nucl. Phys. **B476** (1996) 329, hep-th/9604002.
- [5] W. Hollik, E. Kraus, D. Stöckinger, *Renormalization and symmetry conditions in supersymmetric QED*, Eur. Phys. J. **C11** (1999) 365, hep-ph/9907393.
- [6] W. Hollik, E. Kraus, D. Stöckinger, *Renormalization of supersymmetric Yang-Mill theories with soft supersymmetry breaking*, hep-ph/0007134.
- [7] C. Rupp, R. Scharf, K. Sibold, *Supersymmetry Transformation of Quantum Fields*, hep-th/0101153
- [8] L.V. Avdeev, O.V. Tarasov, A.A. Vladimirov, *Vanishing Of The Three Loop Charge Renormalization Function In A Supersymmetric Gauge Theory* Phys. Lett. **B 96** (1980) 94.